FRACTION ADDITION THROUGH THE MUSIC

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This study examined the effects of an academic intervention, associated with music, on the conceptual understanding of musical notation and arithmetic of fractions of first-year students of high school from a mixed Spanish multicultural and socioeconomic public school. The students (N = 12) had previous concepts about musical instruction, as well as operations with fractions, particularly addition. This is an observational study in which a battery of four tasks was administered before and after an instruction based on a musical environment, music being a semiotic function. The instruction included 9 sessions of 50 minutes each. The results prior to the intervention show deficiencies in a concept that was not new to the students, however, after the intervention the students were competent in addition with fractions.

Keywords: addition of fractions, semiotic function, music, multidisciplinary intervention.

Introduction

Although there is the idea that music and mathematics are very different subjects, the reality is very different. In ancient Greece, Pythagoras (ca. 580-500 BC) realized the relationship between different sounds and the proportion of the strings that emitted them. This led to a description of music using mathematical symbols (Henning, 2009; Weber, 1991, cited in Mall et al., 2016). But the relationship between mathematics and music does not occur because they are close subjects, but "because musical parameters can be transformed and organized using mathematical techniques, and vice versa" (Mall et al., 2016, p. 12). Thus, these are two disciplines in which two systems of compatible signs follow one another: musical notation and numbers (natural and rational). Currently, these systems interact creating a semiotic set (Arzarello, 2006). With all this, interdisciplinary projects acquire great relevance, in particular, projects in which music and mathematics converge.

This context defines the need to continue research on the incorporation of music and its characteristics in the teaching and learning process of mathematics, and vice versa. In particular, with the present work the objective is to evaluate the influence of music, regarding the value of musical notes, in the learning process of the addition of fractions in first grade students of high school (ages between 12 and 14 years old).

Background and frame of reference

According to Avdulova (2018, cited in Veraksa et al., 2022), symbolic play represents the child's ability to link concrete experience with abstract thought by connecting the meaning with the signifier (Piaget, 1962). In this process, the child develops the semiotic function, that is, an ability to represent an absent object or an event that is not directly perceived through symbols or signs (Veraksa et al., 2022).

In the modern teaching of mathematics, the interaction between perception and action also

becomes increasingly important. Different levels of communication work together in semiotic sets (Arzarello, 2006) and students use the cycles of action and perception to develop mathematical understanding. The identification of patterns and their interpretation as a system of signs constitutes a fundamental mathematical task. The repetitions, and therefore the regularities, can be found by observing written symbols. These regularities are the basis of mathematical understanding (Mall et al., 2016).

In this context, the use of musical notation as a tool to acquire the concept of fraction and, in particular, the addition of these numbers, is within what is called addition using discrete models. The school tradition advocates a teaching-learning process of fractions based on continuous models (Rico, 1997), but separating them from a discrete context (Kieren, 1993) means that students are not able to function in contexts in which other meanings or subconstructs of the fraction underlie, for example, as measure, ratio, or operator.

With respect to adding fractions, research has shown that many students learn operations with fractions through memory-based, procedural-oriented instruction, attaching little meaning to such operations and thus making serious errors over time (Behr et al., 1992; Cramer and Henry, 2002; Cramer et al., 2002; Kennedy and Steve, 1997). The Rational Number Project Curriculum is committed to learning fractions through everyday life contexts that are reflected in word problems. Others like Braithwaite et al. (2017) bet on contexts such as the number line; or there are those who see musical arithmetic as a basis for the teaching-learning process of fractions (Oshanova et al., 2022).

Continuing with the last idea, and given that the literature is extensive on the use of music as a multimodal approach for the instruction of fractions (Azaryahu et al., 2020; Courey et al., 2012; Lovemore et al., 2022) in the present work seeks to answer, how does the value of musical notes improve competence in the addition of fractions of students in the first year of high school?

Methodology

This work is part of an experimental and observational study. Within observational studies, this is defined as a short-term longitudinal study. We have worked on the same sample, students belonging to the first grade of high school of the Spanish educational system, during a period of 2 months, obtaining data at the beginning and at the end of an intervention.

This section gives rise to three subsections in which the sample studied, the method used, and finally, the instrument and the research variables are presented.

Sample

Regarding the study participants, it is a small group of 12 students belonging to a public high school in Valencia, Spain. Among the students there is a high number of immigrants and students at risk of school absenteeism. In turn, there is great diversity with respect to the economic and cultural levels of the families of the students that make up the classroom. The ages of the students are between 12 and 14 years old.

Method

The study has three different sections. First, a battery of four questions was asked, prior to the intervention (pre-test). After this, an intervention is carried out with an approximate duration of two months (9 sessions of 50 minutes each). Finally, a second battery of four questions (post-test) is carried out. Both batteries of questions contain questions related to the contents of fractions that the students have already seen in the previous course (6th of primary education) and that will be extended in their present course.

This paper is part of a broader study that covers the concepts of fractions, representation,

ordering, equivalence, and fraction arithmetic. Due to the extension allowed, only results related to the arithmetic of fractions, in particular addition, are presented here. In the proposed intervention, it is the third, fifth and sixth sessions that focus their content on this specific concept; which are detailed below.

Session 3. The main purpose of this session is to explain the addition and subtraction of fractions with the same denominator using musical elements. In his case, the manual (Adiel, 2021), is limited to refreshing the algorithmic process of this operation. In addition, it introduces a graphic example using a continuous model based on the area model (Figure 1).

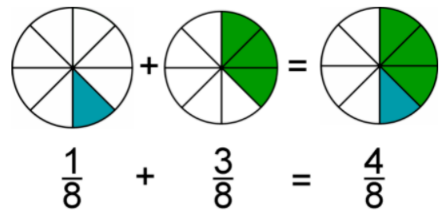

Figure 1: Example of graphic sum (Adiel, 2021)

To this aspect, in the intervention carried out, two graphic ways of carrying out or verifying these operations from a musical field are exposed: a linear one and a discrete one. To exemplify the linear model, the strings of a cello are used (Figure 2).

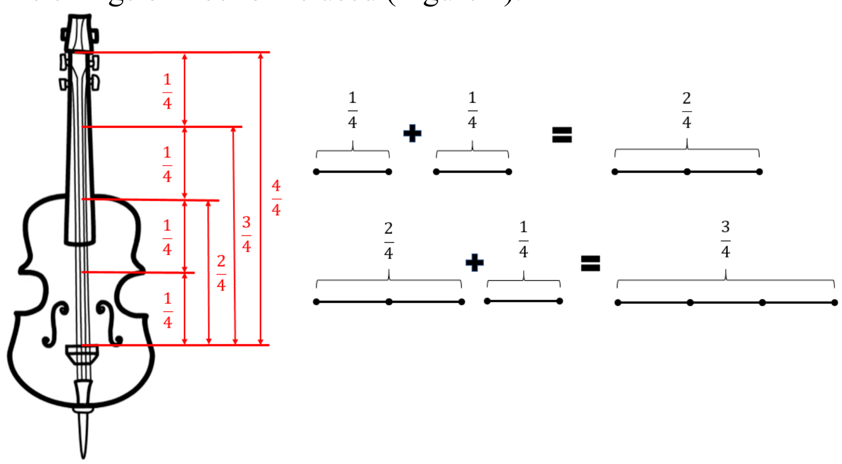

Figure 2: Addition of fractions linear model, cello strings

Instead, to exemplify the discrete model, musical notes are used (Figure 3).

Figure 3: Addition and subtraction of fractions with discrete model

Session 5. In this session the addition and subtraction of fractions is worked on again, but with the particularity that the denominator differs. As in the previous session, the graphic support provided by the musical representation continues to be the element that promotes the achievement of the main objective stated at the beginning of the document.

As a practical example, the students are presented with the one shown in Figure 4. The aforementioned figure shows the process followed to add two fractions with different

denominators and, at the same time, the graphic representation from the musical perspective that is has been using in previous sessions, specifically with the use of the linear model.

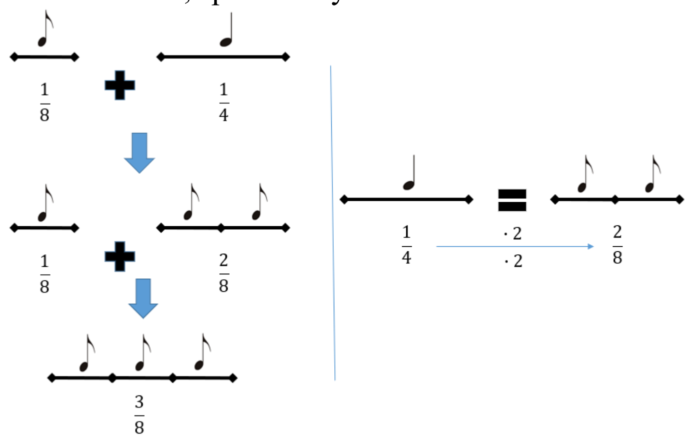

Figure 4: Addition of fractions with different denominators

Session 6. The main objective of this session is to propose to the students a more realistic situation, in the musical field, in which to make direct use of the addition of fractions.

For this purpose, an extract of four bars from Symphony No. 9, the Ode of Joy (van Beethoven, 1824), commonly called Hymn of Joy (Figure 5), is used to show how the discrete representation model exposed in the previous sessions is not an artificial construction. The students verify that this codification that has been presented to them previously is, in reality, used by composers and performers in their day to day. The students will observe that the compass is nothing more than the sum of the values of the notes located between the dividing lines.

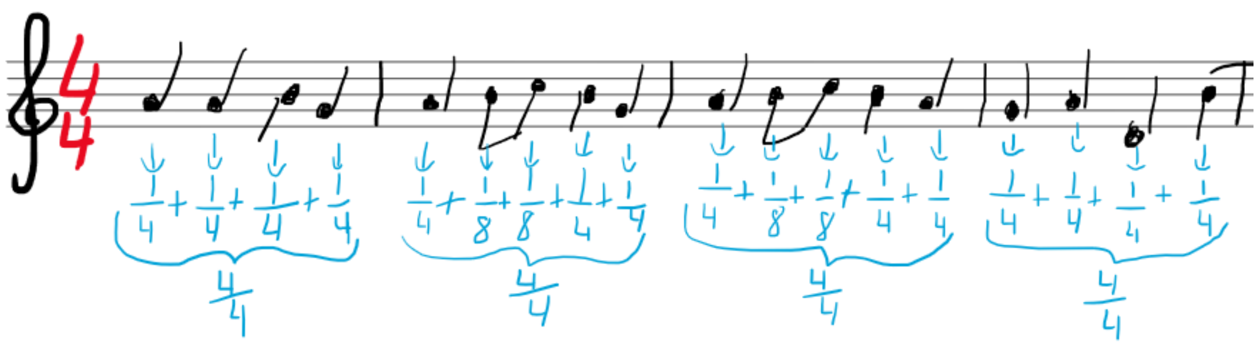

Figure 5: Hymn of Joy extract, addition of fractions

Likewise, they will realize that they themselves make use of this encoding during their music classes at the center. It is hoped, then, that at this point they will see a manifest utility of fractions. Utility of which they themselves were users without knowing it, thus creating that contextualization that arouses their attention and interest.

Instrument and Research Variables

To obtain research data, two pencil and paper batteries with 4 questions each are used. Although some data changes in the questions between the pre-test and the post-test, the isomorphism between the two is preserved to make possible an evaluation of the knowledge evolution of the students.

Q1. Represent, in as many ways as possible, the following fraction: ¼

Q2. Order, from smallest to largest:

Pre-test: 2/2,1/4, 7/8, 5/4. Post-test: 1/2, 3/4, 2/8, 2/2
Q3. Write and represent two equivalent fractions.
Q4. Calculate, you can rely on representations:
Pre-test: a) 1/2+3/8; b) 2/2-1/4. Post-test: a) 2/4+1/2, b) 6/8-1/4.

Following the objective and question of this report, the focus is on the addition of fractions, so only the analysis and research variables of Q4 will be determined.

The question Q4 allows to check the abilities in the addition of fractions of the students. In particular, it allows analyzing whether they use alternative methods, such as representation, which are suggested to them, or only apply the algorithm of said operation. On the other hand, possible errors made in carrying out this process will also be highlighted, if any.

In order to carry out a descriptive analysis of this issue, the research variables defined are three:

- Success: relationship between students who have answered the question satisfactorily with respect to the total.
- Process: this field introduces the method used to solve the question: operate with the decimal expression (ED), use the graphical representation (RP) or apply the algorithm (AL).
- Mistakes: use of the multiplication algorithm (ALmult), use of the division algorithm (ALdiv), common denominator (CD).

Results and discussion

First, a descriptive analysis of the results obtained in the pre-test is carried out, followed by the analysis of the results obtained in the post-test. Subsequently, the evolution of the students globally is presented and ends with the individual evolution.

Q4 Results

As can be seen in the pre-test column, in Table 1, only one student out of 12 has managed to successfully add fractions.

This is in line with what is obtained in previous studies (Braithwaite, 2017) in which it is determined that students between the ages of 13 and 14 continue to have difficulties when performing this operation.

In the same column it can also be verified that the method used in the pre-test, by the student who has solved correctly (ED, 1/12) has not involved the manipulation of the fractions, since the student has obtained the expression decimal and has operated with these expressions (Carpenter et al., 1976) (see Figure 6).

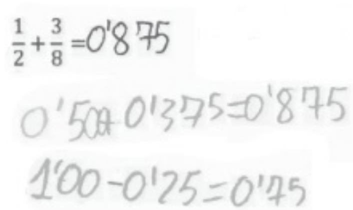

Figure 6: Decimal Expression Method

Although this is a perfectly valid strategy, it also confirms that at the time of the pre-test the students did not remember the tools to answer the question by operating on the fractions, a fact that is evident in the errors made.

Table 1: Q4 Results

	Pretest	Post Test
Success	1/12	10/12

Method	ED	1/12	-
	RP	-	1/12
	AL	-	9/12
Null		3/12	-
Mistake	ALmult	7/12	-
	ALdiv	1/12	1/12
	CD	0	1/12

It is determined in the pre-test a general tendency on the part of the students to commit the same type of error, confusion with the rule of the algorithm of multiplication of fractions (7/12). The students apply the Multiplication algorithm, but adapted to addition, thus adding numerator with numerator and denominator with denominator (Figure 7) (Howard, 1991; Keller et al., 1940; NAEP, 2013).

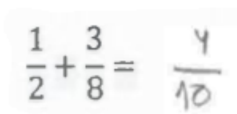

Figure 7: Errors when applying the addition algorithm, use of the Multiplication Algorithm

Another error marked by previous studies (Mata and Porcel, 2006) is the adaptation of the use of the Division Algorithm. Thus, the student who uses it in the pre-test performs a cross addition (Figure 8).

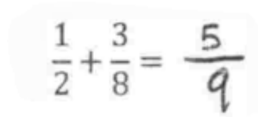

Figure 8: Errors when applying the addition algorithm, use of the Division Algorithm

This confusion in the rules of the algorithms verifies the fact that the rule is rote learning, which implies that over time it is forgotten or confused with similar rules (Hart, 1981).

Analyzing now the results obtained after the intervention -post-test-, a significant increase in success can be observed (10/12), which leads to determine that the use of music as a methodology has favored the teaching-learning process. (Azaryahu et al., 2020; Courey et al., 2012; Lovemore et al., 2022). This statement is highlighted, since the students who have participated in this study, had previous knowledge in the sum of fractions, because it is a knowledge, which in the Spanish curriculum begins in 4th grade of primary pducation and is repeated in 5th and 6th grade. of this primary level, which would indicate that their knowledge should be consolidated in the first year of secondary education, which, as has been verified, the results in this study provide other information.

If the methods used to solve the activity are analyzed, it can be observed that the application of the fraction addition algorithm is the process most used by the students (9/12), being a student the one who has chosen to solve the operations, exceptionally graphically (Figure 9) (Rau et al., 2015).

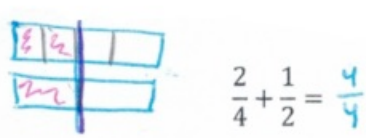

Figure 9: Use of the Graphical Representation in the addition of fractions

Finally, regarding the errors in the post-test, it should be noted that a student presented

confusion in the use of the division algorithm. This student, as can be seen in Table 2 of the following section, had made an error by applying the multiplication algorithm. In addition, an error appears related to making the common denominator (Vinner et al., 1981) (Figure 10).

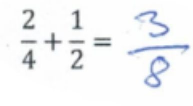

Figure 10: Errors when applying the addition algorithm, Common Denominator

Evolution Q4

Regarding the evolution (Table 2), first of all, the significant increase in students who have performed both operations after the intervention stands out, going from one to ten students.

Table 2: Individual evolution Q4

Student	Pretest			Post Test		
	Success	Method	Mistake	Success	Method	Mistake
S1,S2,S3, S4,S7,S12	0	AL	ALmult	1	AL	-
S9	0	AL	ALmult	1	RG	-
S10	0	AL	ALmult	0	AL	ALdiv
S16	0	AL	null	0	AL	CD
S15	0	AL	null	1	AL	-
S6	0	AL	ALdiv	1	AL	-
S13	1	ED	-	1	AL	-

Regarding the methods used by the students in the resolution, in the pre-test, the only student who carried out the operations did so using the decimal expression of the proposed fractions, as explained in detail above. On the other hand, in the post-test it is observed that no student makes use of this strategy and, instead, all of them operate directly with the fractions and there is even a student who performs the operations graphically (Figure 9).

Another positive aspect is that the blank answers in the post-test disappear, the two students (S15 and S16) go to an algorithmic resolution, one correct (S15) and the other with an error in obtaining the common denominator.

Continuing with the errors, the use of the division algorithm persists, although it is not maintained by S6 who does so in the pre-test, but it is the student S10 who goes from using the multiplication algorithm to that of division.

Conclusion

The present observational study shows that students between 13 and 14 years of age have significant deficiencies in the addition of fractions, despite being a mathematical concept that has been present in the Spanish curriculum since the age of 10 (LOMLOE, 2022). However, an interdisciplinary intervention, in which a system of musical signs is used as a base as a semiotic function to try to improve skills with fractional numbers, particularly with the addition of fractions, allows reversing a dramatic initial situation.

The results presented here are part of an investigation in which, in addition to the addition of fractions, the difficulties that students have in graphic representation, ordering and equivalence of fractions are analyzed, and how music allows solving part of the problems. initial deficiencies. Despite the fact that, in the addition of fractions, a direct consequence of the intervention carried

out is not observed, it is obtained in the rest of the concepts evaluated, in particular, in the equivalence of fractions, where the students make use of the system of musical signs, to solve the situations raised. This situation is highlighted, since as indicated by Vinner et al. (1981) for the sum of fractions there are two ideas that must be present, a common denominator and equivalent fractions, the second idea being assumed by the students who have participated in this research.

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